Spirited Energy — Mathematical Addendum

PUB Companion: Dual-Ledger Firm Capacity, Economics, and Water Arithmetic

Companion to the voter-facing master report
Canonical registry: `research/assumptions.yaml`
System B: `research/system_b_ledger.py`
12 August 2026

Abstract

Audience. This addendum is written for Public Utilities Board staff, advisors, and independent auditors. The master report (`spirited-energy-master-report.pdf`) explains the hybrid plan in plain speech for voters. This file states every governing equation with numerical substitution from the machine-readable registry. Thermal power (MWth) and electric power (MW) never share a headline sum. Evidence classes: O official / C computed & tested / E engineering estimate / A needs confirmation / PD public disclosure — verify on docket.

1 Division of labour

Master report (voters)	**This addendum (PUB)**
Hook line; cold morning; twin $3B choice	Dual-ledger axiom and retired composites
Five hybrid pieces; winter modelling charts	Accreditation substitutions 413 / 688 / 788
Summer PDRC / BTES as *added benefit*	Summer equations (separate; not winter firm)
All-the-numbers table; cabinet ask	$/kW matrix; 20-year band gates; pass predicates
Falsifiers in prose	ELCC ask formalised; reproduction commands

Do not treat the narrative PDF as a substitute for these equations. Do not treat these equations as a substitute for the civic ask (model sewage heat vs gas; taps at NEWPCC). Do not add summer cooling megawatts into winter firm accreditation.

2 Notation

- P: electric power (MW). Q: thermal power (MWth).
- C: capital ($). Subscripts: g gas, h hybrid.
- $S = 600\,\text{MW}$: 2030 firm shortfall [PD].
- $P_{\text{peak}} = 5096\,\text{MW}$: January 2024 system peak [PD].

3 Portfolio map (building blocks)

The voter report lists five tools. Mapping to symbols used below:

Voter piece	Symbol	Notes
Sewage / heat hub	Q_{hub}, $P_{\text{hub,net}}$	Thermal ledger; net electric credit only
Batteries (4 h)	P_{BESS}	300 MW / 1200 MWh
Demand response	P_{DSM}	150 MW; half in conservative
Interties / dual-fuel backup	P_{intertie}	Cap 300 MW; deliverability [A]
Public compute (gated)	$\subset Q_{\text{compute}}$	10–20 kW nodes; eligible = 0 today [C]

4 Sewage-heat physics (effluent stage) [E]

Daily NEWPCC flow $\dot{V} \approx 195 \times 10^6$ L/d. Cooling treated effluent by ΔT recovers source heat

$$Q_{\text{src}}(\Delta T) \approx 9.4\,\text{MWth} \cdot \Delta T\,[^\circ\text{C}], \tag{1}$$

so $Q_{\text{src}}(1\,^\circ\text{C}) \approx 9.4$ MWth and $Q_{\text{src}}(4\,^\circ\text{C}) \approx 38$ MWth (river-safe standard practice). Industrial heat pumps lift to district temperature at a delivered COP ≈ 4.3 [E] (heat delivered per unit electric input; equivalently ≈ 3.3 units of source heat moved per unit of work — a low-lift sewage-source machine). Delivered effluent-stage heat is therefore $Q_{\text{eff}} = Q_{\text{src}}(4\,^\circ\text{C}) + P_{\text{comp}} \approx 37.6 + 12 \approx 50$ MWth, inside the river-safe draw, and the electric credit for the effluent stage is $P_{\text{hub,eff}} = 38$ MW (see (12)). False Creek / Markham establish equipment availability; timing risk is contractual (taps before last major NEWPCC contracts), not physics.

5 Summer add-on (NOT winter firm) [E]

Manitoba's capacity fight is winter-peaking. The following summer levers are *residual benefits* of the same stewardship stack. They are registered in `assumptions.yaml` and gated so they never enter P_{cons}, P_{ref}, or P_{full}.

Passive daytime radiative cooling on an illustrative rooftop envelope. The film rejects a *thermal* flux; the avoided *electric* A/C load is that flux divided by the chiller COP it displaces:

$$A_{\text{PDRC}} = 2{,}400{,}000\,\text{m}^2, \qquad q_{\text{net}} \approx 75\,\text{W/m}^2, \tag{2}$$

$$Q_{\text{PDRC}} = A_{\text{PDRC}} \cdot q_{\text{net}} \cdot 10^{-6} = 180\,\text{MWth (rejected heat)}, \tag{3}$$

$$P_{\text{PDRC}} = Q_{\text{PDRC}}/\text{COP}_{\text{chiller}} \approx 180/3.5 \approx 50\,\text{MW (avoided electric A/C load)}. \tag{4}$$

Heat Host / BTES chiller avoidance (parasitic cooling not run while reject heat is injected to seasonal storage):

$$P_{\text{BTES}} = 70\,\text{MW (summer only)}. \tag{5}$$

Combined summer residual:

$$P_{\text{summer}} = P_{\text{PDRC}} + P_{\text{BTES}} = 120\,\text{MW}, \qquad \boxed{P_{\text{summer}} \notin \{P_{\text{cons}}, P_{\text{ref}}, P_{\text{full}}\}}. \tag{6}$$

Illustrative summer heatwave peak band: 3800–4200 MW [E].

6 Dual-ledger axiom (non-negotiable)

$$\boxed{P_{\text{firm}} \;\neq\; P_{\text{elec}} + Q_{\text{th}}} \tag{7}$$

The retired anti-pattern summed thermal into electric:

$$P_{\text{retired}} = P_{\text{BESS}} + (Q_{\text{effluent}} + Q_{\text{compute}}) + P_{\text{DSM}} + P_{\text{intertie}} \tag{8}$$

$$= 300 + (50 + 95) + 150 + 300 = 895\,\text{MW}. \tag{9}$$

Equation (9) is **forbidden** as a firm claim. Tests enforce $P_{\text{headline}} \neq 895$.

7 Thermal ledger

The hub that counts today is the effluent stage only:

$$Q_{\text{hub}}^{\text{today}} = Q_{\text{effluent}} = 50\,\text{MWth (delivered)}. \tag{10}$$

Contingent compute reject heat, $Q_{\text{compute}} = 95\,\text{MWth}$, stays behind the three gates and is *never* bookable as firm winter supply: reject heat only exists while its IT load is drawing power, so any honest winter credit nets the load first (§below). Compressor draw for the effluent stage is $P_{\text{comp}} \approx 12\,\text{MW}$ at delivered COP ≈ 4.3 [E].

8 Hub net electric (displacement accounting)

Stated assumption: displaced load is electric *resistance* heating (COP = 1) on the coldest mornings. If the displaced stock were air-source heat pumps at COP $\in [1.5, 2]$ in the tail hours, the credit scales down proportionally ($38 \to 19$–$25\,\text{MW}$); the scenarios below carry the resistance case and label it.

$$P_{\text{hub,net}} = Q_{\text{hub}}^{\text{today}} - P_{\text{comp}} = 50 - 12 = 38\,\text{MW}. \tag{11}$$

Effluent-only stage (identical; kept for stage naming):

$$P_{\text{hub,eff}} = 38\,\text{MW}. \tag{12}$$

Always $P_{\text{hub,net}} < Q_{\text{hub}}^{\text{today}}$ (pass gate).

Why compute reject heat is not a firm-supply credit. Netted honestly on the same basis, contingent compute at full build would add $+95$ displaced $-\,22$ compressor $-\,95$ new IT load $\approx -22\,\text{MW}$: a net *liability* in the peak hour if run flat. The design answer is curtailment: compute pauses in the top load hours while its banked reject heat (BTES) keeps serving the district loop — demand response with a thermal flywheel, entering the ledger only through the DSM line once contracted, never as hub supply.

9 Electric building blocks

$$P_{\text{BESS}} = 300\,\text{MW}\ (4\,\text{h} \Rightarrow 1200\,\text{MWh}), \tag{13}$$

$$P_{\text{DSM}} = 150\,\text{MW}, \tag{14}$$

$$P_{\text{intertie}}^{\text{cap}} = 300\,\text{MW (deliverability pending [A])}. \tag{15}$$

10 Accreditation scenarios [C]

$$\begin{aligned} P_{\text{cons}} &= P_{\text{BESS}} + \tfrac{1}{2}P_{\text{DSM}} + 0 \cdot P_{\text{intertie}} + P_{\text{hub,eff}} \\ &= 300 + 75 + 0 + 38 = 413\,\text{MW}, \end{aligned} \tag{16}$$

$$\begin{aligned} P_{\text{ref}} &= P_{\text{BESS}} + P_{\text{DSM}} + 200 + P_{\text{hub,net}} \\ &= 300 + 150 + 200 + 38 = 688\,\text{MW}, \end{aligned} \tag{17}$$

$$\begin{aligned} P_{\text{full}} &= P_{\text{BESS}} + P_{\text{DSM}} + P_{\text{intertie}}^{\text{cap}} + P_{\text{hub,net}} \\ &= 300 + 150 + 300 + 38 = 788\,\text{MW}. \end{aligned} \tag{18}$$

Shortfall coverage and margins:

$$M_{\text{s}} = P_{\text{s}} - S, \qquad M_{\text{cons}} = -187,\ M_{\text{ref}} = 88,\ M_{\text{full}} = 188\ (\text{MW}). \tag{19}$$

Reference and full cover S ($M \geq 0$); conservative does not — by design.

Illustrative ELCC haircut (applied to our own resources first). Accrediting 4-hour BESS at 85% and DSM at 80%: $0.85 \cdot 300 + 0.80 \cdot 150 + 200 + 38 = 613\,\text{MW} \geq S$. The reference case survives the same treatment we ask the Board to apply to the gas plan. Note the margin above the conservative floor is carried by interties tagged [A]; that dependency is disclosed, not hidden.

11 Gas plan (comparison)

$$P_{\text{g,name}} = 750\,\text{MW}, \qquad C_{\text{g}} = \$3.00\,\text{B}, \tag{20}$$

$$P_{\text{g,derate}}^{\text{scen}} = 525\,\text{MW}\ \text{(event-informed scenario [A], not a proven ELCC)}. \tag{21}$$

Cold-event record: 3 of 10 highest-demand winter events [PD]. That frequency is an *ELCC study ask*, not a flat derate asserted as fact:

$$\frac{3}{10} \;\not\Rightarrow\; 0.30 \cdot P_{\text{g,name}}\ \text{as finding}. \tag{22}$$

12 Delivered capital intensity

$$c_{\text{kW}}(P) = \frac{C}{P \cdot 10^3} \quad (\$/\text{kW}). \tag{23}$$

Hybrid capital:

$$C_{\text{h,gross}} = \$1.95\,\text{B}, \qquad C_{\text{ITC}} = \$0.45\,\text{B}\ [A], \tag{24}$$

$$C_{\text{h,net}} = C_{\text{h,gross}} - C_{\text{ITC}} = \$1.50\,\text{B}. \tag{25}$$

Substituting (23):

Scenario	P (MW)	Gross \$/kW	Net \$/kW
conservative	413	4,722	3,632
reference	688	2,834	2,180
full	788	2,475	1,904
gas nameplate	750	4,000	—
gas derate scen.	525	5,714	—

Pass gate: $c_{\mathrm{kW}}^{\mathrm{h,gross}}(P_{\mathrm{ref}}) < c_{\mathrm{kW}}^{\mathrm{g,name}}(P_{\mathrm{g,name}})$ — gross against *nameplate*, with no ITC and no derate assumed. The derate scenario column is a sensitivity only, never the comparator of record.

13 Twenty-year cost bands

Registry bands (driver-attributed corners [E/A]):

$$C_{\mathrm{g}}^{(20)} \in [4.5,\, 6.9]\,\mathrm{B}, \tag{26}$$

$$C_{\mathrm{h}}^{(20)} \in [1.5,\, 1.8]\,\mathrm{B}. \tag{27}$$

The hybrid band is stated *net of the ITC*; on a gross basis (ITC add-back +\$0.45B) the band is $[1.95,\, 2.25]\,\mathrm{B}$ and the savings band becomes $[2.25,\, 4.95]\,\mathrm{B}$ — the conclusion is unchanged. Line-item O&M, DSM payments, loop operation and BESS augmentation are itemised in the forensic accounting whitepaper. Non-overlap gate: $C_{\mathrm{h,high}}^{(20)} < C_{\mathrm{g,low}}^{(20)}$. Savings band:

$$\Delta_{\mathrm{lo}} = C_{\mathrm{g,low}}^{(20)} - C_{\mathrm{h,high}}^{(20)} = 2.7\,\mathrm{B}, \tag{28}$$

$$\Delta_{\mathrm{hi}} = C_{\mathrm{g,high}}^{(20)} - C_{\mathrm{h,low}}^{(20)} = 5.4\,\mathrm{B}. \tag{29}$$

14 Hourly peak-shave profile [C]

Let $h \in \{0, \ldots, 23\}$. Hub contributes constant net electric MW; BESS+DSM add on dual peaks:

$$P_{\mathrm{shave}}(h) = P_{\mathrm{hub,net}} + \begin{cases} P_{\mathrm{BESS}} + P_{\mathrm{DSM}}, & h \in [7,9] \cup [17,19], \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

15 Eligibility gates [C]

$$P_{\mathrm{eligible}} = \begin{cases} P_{\mathrm{IT}}, & \text{gates 1–3 all true}, \\ 0, & \text{otherwise.} \end{cases} \qquad P_{\mathrm{eligible}}^{\mathrm{today}} = 0\,\mathrm{MW}. \tag{31}$$

Locked node band: $P_{\mathrm{node}} \in [10, 20]\,\mathrm{kW}$. Hyperscale equivalence at 250–300 MW requires $N \in [12500, 30000]$ nodes [C].

16 Phosphorus honesty [E/O]

$$R_{\mathrm{surplus}} \in [15, 320]\,\mathrm{t\,P/yr} \text{ (scales with node count)}, \tag{32}$$

$$R_{\mathrm{stack}} \approx 1260\,\mathrm{t\,P/yr} \text{ (full multi-measure mid-case)}, \tag{33}$$

$$G_{\mathrm{ext}} \approx 3050\,\mathrm{t\,P/yr}, \qquad T_{\mathrm{border}} = 1400\,\mathrm{t\,P/yr}\ [O]. \tag{34}$$

Governance floor: $\geq 35\%$ of distributable surplus to remediation as first lien [E]. Claim: $R_{\mathrm{surplus}} \ll G_{\mathrm{ext}}$; the energy plan does not close the lake gap alone.

17 System B pass predicates

$$P_{\text{full}} = P_{\text{BESS}} + P_{\text{DSM}} + P^{\text{cap}}_{\text{intertie}} + P_{\text{hub,net}}, \tag{35}$$

$$P_{\text{ref}} \neq 895, \quad P_{\text{ref}} \geq S, \quad P_{\text{full}} \geq S, \tag{36}$$

$$c^{\text{h,gross}}_{\text{kW}}(P_{\text{ref}}) < c^{\text{g,name}}_{\text{kW}}(P_{\text{g,name}}), \tag{37}$$

$$C^{(20)}_{\text{h,high}} < C^{(20)}_{\text{g,low}}, \quad P_{\text{hub,net}} < Q_{\text{hub}}, \tag{38}$$

$$P_{\text{summer}} \notin \{P_{\text{cons}}, P_{\text{ref}}, P_{\text{full}}\}. \tag{39}$$

All predicates currently **PASS** under `build_ledger().pass_report()`.

18 Reproduction

```
python3 research/recalculate_and_generate_all.py
python3 research/scripts/generate_math_addendum.py
python3 research/scripts/verify_system_ab_alignment.py
```

Source of truth: `assumptions.yaml`. This PDF is generated; do not hand-edit numbers. Narrative companion: `report/spirited-energy-master-report.pdf` (voter hybrid primer, winter modelling, summer add-on, Board ask).